April 30, 2016

Problems for Quiz 2

The quiz next friday, May 6, will consist of five of the following problems.

As usual, you will be expected to justify your assertions.

1. Let X be the affine surface in $\mathbb{A}^3$ defined by the equation $x_1^3+x_1x_2x_3+x_1x_3+x_2^2+x_3=0$, and let $\overline{X}$ be its closure in $\mathbb{P}^3$. Describe the intersection of $\overline{X}$ with the plane at infinity in $\mathbb{P}^3$.

2. Let Y be a smooth curve with function field K, and let (a_0, a_1, a_2) be a point of $\mathbb{P}^2$ with values in K, i.e., with $a_i \in K$ not all zero. Explain how to extend this point to a map $Y \to \mathbb{P}^2$.

3. Prove that a valuation ring is a unique factorization domain.

4. Let Y be the subvariety of affine space $\mathbb{A}^n$ of zeros of a prime ideal P of $\mathbb{C}[x_1, ..., x_n]$, and suppose that P is generated by two elements f, g. What can said about the dimension of Y?

5. Prove that if a subset S of a variety X is constructible in the Zariski topology and is closed in the classical topology, then it is closed in the Zariski topology.

6. Let $X = \mathbb{P}^2$, and let U be the complement of the point $(1, 0, 0)$ in X. Determine the sections $\mathcal{O}_X(U)$ of the structure sheaf on U.

7. Let Y be the surface in $\mathbb{P}^3$ defined by an irreducible polynomial of degree 4. Determine the dimensions of the cohomology groups $H^q(Y, O_Y)$.

8. Let $A \subset B$ be finite-type domains. Prove that if B is a finite A-module, the inclusion induces a surjective map $\operatorname{Spec} B \to \operatorname{Spec} A$.

9. Let $X = \mathbb{P}^2$. What are the sections of the twisting module $\mathcal{O}_X(n)$ on the open complement of the line $x_0 + x_1 + x_2 = 0$?

10. Let α be an element of a domain A, and let $\beta = \alpha^{-1}$. Prove that if β is integral over A, then β is an element of A.