April 6, 2016

18.721 Assignment 7

This assignment is due Friday, April 15.

I hope you like this assignment. When you have done it, you will have learned the important parts of Chapter 5.

1. A projective variety if a closed subvariety of $\mathbb{P}^n$. A quasiprojective variety is an open subset of a projective variety. Prove the following converse to Theorem 5.9.4: If a quasiprojective variety X is proper, then it is a projective variety.

2. With notation as in Example 5.9.5, prove that the set of reducible divisors of degree d in $\mathbb{P}^2$ form a closed subset of the space Z of all divisors of degree d.

3. Prove Theorem 5.10.2 on semicontinuity of fibre dimension. You will want to use
   Krull's Theorem,
   the fact that maximal ideals of a smooth affine curve are locally principle,
   Proposition 5.2.5 extending the Noether Normalization Theorem,
   the method of proof of Theorem 5.6.5, and
   the Curve Criterion for closed sets.

PS. The original version of the notes for Chapter 5 contained a proof. I've deleted it, but in case you printed the chapter out, please try to do the problem without looking it up.