Spring 2016

18.721 Assignment 1

This assignment is due friday, February 14

1. Let $f(x,y,z)$ be an irreducible homogeneous polynomial of degree > 1. Prove that the locus $f=0$ in $\mathbb{P}^2$ contains three points that do not lie on a line.

2. Let f and g be irreducible homogeneous polynomials in x,y,z. Prove that if the loci $\{f=0\}$ and $\{g=0\}$ are equal, then $g=cf$.

3. Prove that a smooth point of a curve is a flex point if and only if the Hessian determinant is zero, in the following way: Given a smooth point p of X, choose coordinates so that $p=(0,0,1)$ and the tangent line ℓ is the line $\{x_1=0\}$. Then compute the Hessian. To complete the proof, verify that the vanishing of its determinant isn't affected by a change of coordinates.

4. Prove that a plane cubic curve can have at most one singular point. Do this without using Bézout's Theorem.

5. Compute $\prod_{i\neq j}(\zeta^i-\zeta^j)$ when $\zeta=e^{2\pi i/n}$.

6. Let C be a smooth cubic curve in $\mathbb{P}^2$, and let p be a flex point of C. Choose coordinates so that p is the point $(0,1,0)$ and the tangent line to C at p is the line $\{z=0\}$.
(a) Show that the coefficients of x^2y, xy^2, and y^3 in the defining polynomial f of C are zero.
(b) Show that with a suitable choice of coordinates, one can reduce the defining polynomial to the form $f=y^2z+x^3+axz^2+bz^3$, where x^3+ax+b is a polynomial with distinct roots.
(c) Show that one of the coefficients a or b can be eliminated.